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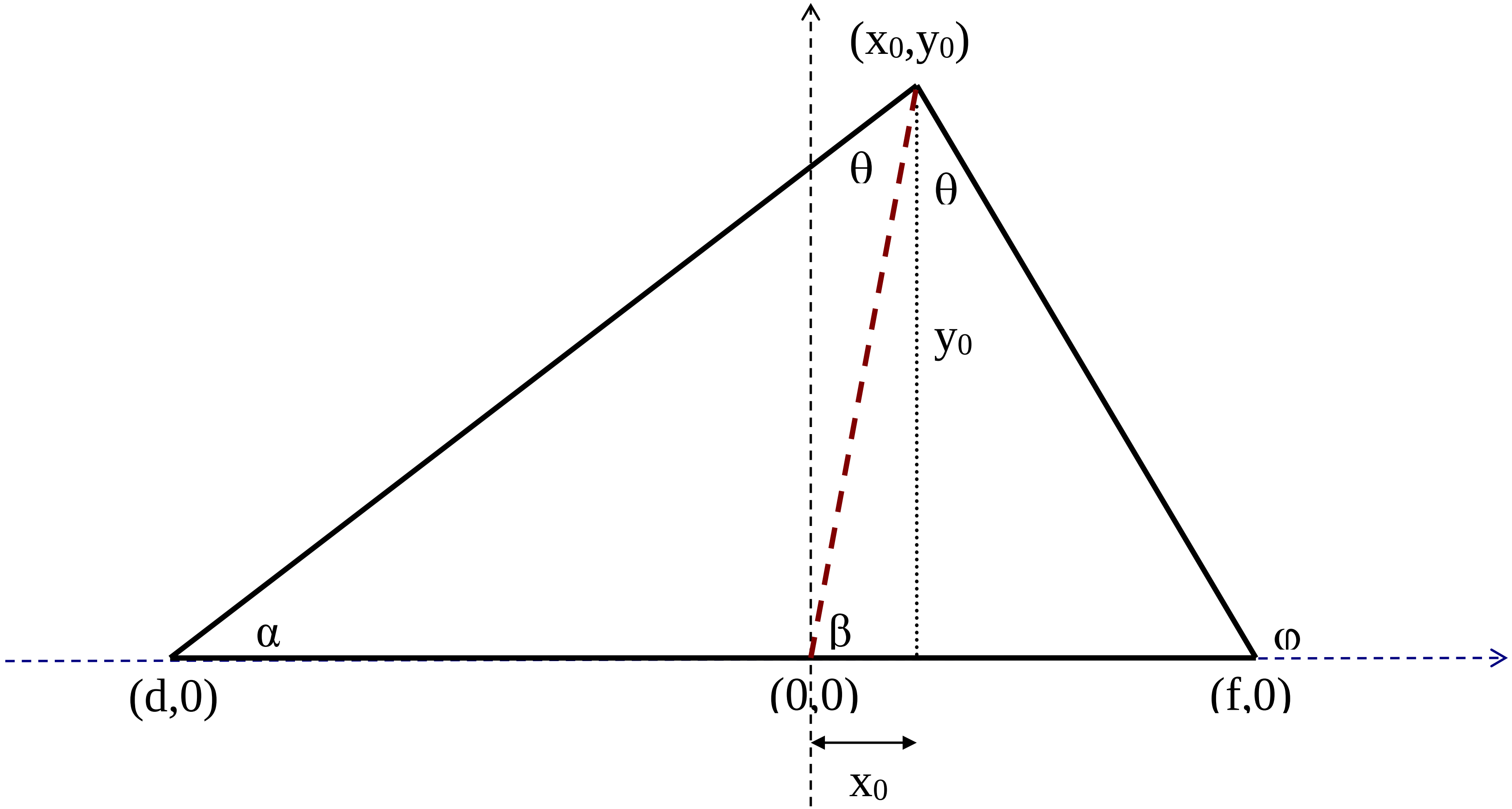

The triangle in figure is typical acute one i.e. the exterior angle φ is larger than 90º. Here the base (of the altitude y_0) is chosen to be on the x-axis. The bisector of the vertex angle

(the red dashed line) intersects with the base at the origin point (0,0) of the Cartesian x-y coordinates. It's obvious that the tangents of the angles φ, β and α have the same numerator y_0 but this time the interior angle ($180^o - \varphi$) is involved, however this will result symbolically similar equations as for the obtuse triangle.

$\tan(\beta) = y_0 / x_0$, $\tan(\alpha) = y_0 / (x_0 - d)$, d here is negative!

$\tan(\varphi) = -\tan(180^o - \varphi) = -y_0/(f - x_0) = y_0/(x_0 - f)$

Again the base of the triangle consists of two parts f and d and the task is proposed to find in general the relationship between these two parts. Here is the approach!

The exterior angles $\varphi = \beta + \theta$, while **$\beta = \theta + \alpha$,** which gives:

$2\beta = \varphi + \alpha$, and therefore **$\tan(2\beta) = \tan(\varphi + \alpha)$,** by applying the tangent of sum:

$$\frac{2\tan(\beta)}{1 - \tan^2(\beta)} = \frac{\tan(\varphi) + \tan(\alpha)}{1 - \tan(\varphi)\tan(\alpha)}$$

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This equation can be simplified by multiplying the numerator of one side with the denominator of the other side which results another equation:

$[\ 2\tan(\beta)\ ]\ [\ 1 - \tan(\varphi)\tan(\alpha)\ ] = [\tan(\varphi) + \tan(\alpha)]\ [(1 - \tan^2(\beta)]$

Prior continue it is important to be very careful with the +/-- signs. Now substitute the tangents with their values:

$$[2y_0 / x_0]\ [\ 1 - (y_0^2 /(x_0 - f)\ (x_0 - d)\ ] = [y_0/(x_0 - f) + y_0 / (x_0 - d)\ ]\ [1 - (y_0^2 / x_0^2)]$$

We can rewrite the above equation in order to recognize common values:

$$\frac{2y_0}{x_0}\left[1 - \frac{y_0^2}{(x_0 - f)(x_0 - d)}\right] = \left[\frac{y_0}{(x_0 - f)} + \frac{y_0}{(x_0 - d)}\right]\left[1 - \frac{y_0^2}{x_0^2}\right]$$

You can notice that y_0 is common numerator on both sides of the equation which can be eliminated by dividing by y_0 and we get:

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$$\frac{2}{x_0}\left[1 - \frac{y_0^2}{(x_0 - f)(x_0 - d)}\right] = \left[\frac{1}{(x_0 - f)} + \frac{1}{(x_0 - d)}\right]\left[1 - \frac{y_0^2}{x_0^2}\right]$$

Further simplifying:

$$\frac{2}{x_0}\left[\frac{(x_0 - f)(x_0 - d) - y_0^2}{[(x_0 - f)(x_0 - d)]}\right] = \left[\frac{(x_0 - d) + (x_0 - f)}{[(x_0 - f)(x_0 - d)]}\right]\left[\frac{x_0^2 - y_0^2}{x_0^2}\right]$$

Multiplying both sides by $x_0 [(x_0 - f)(x_0 - d)]$ will result:

$$2 [x_0^2 - y_0^2 - (f + d) x_0 + f d] = [2 x_0 - (f + d)] \left[\frac{x_0^2 - y_0^2}{x_0}\right]$$

Again multiplying both sides by x_0 the equation becomes:

$$2 (x_0^2 - y_0^2) x_0 - 2 (f + d) x_0^2 + 2 f d x_0 = 2 (x_0^2 - y_0^2) x_0 - (f + d) x_0^2 + (f + d) y_0^2$$

On both sides the term $2 (x_0^2 - y_0^2)$ can be eliminated and the equation becomes

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$2 f d x_0 = (f + d) x_0^2 + (f + d) y_0^2 = (f + d) (x_0^2 + y_0^2)$

Since $(x_0^2 + y_0^2) =$ the square of the length of the bisector (the red dashed line) which is denoted by the letter r , and the last equation will become more simpler:

$2 f d x_0 = (f + d) r^2$ and can be rewritten:

$2 x_0 / r^2 = (f + d) / f d$, and finally:

$$2 x_0 / r^2 = 1/f + 1/d$$

This means that the same relationship is valid as in the case of obtuse triangle.

Conclusion: the above equation defines generally how the two parts of the base are related to each other when the bisector intersects with the base!!