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Geometrical representation of the reciprocal

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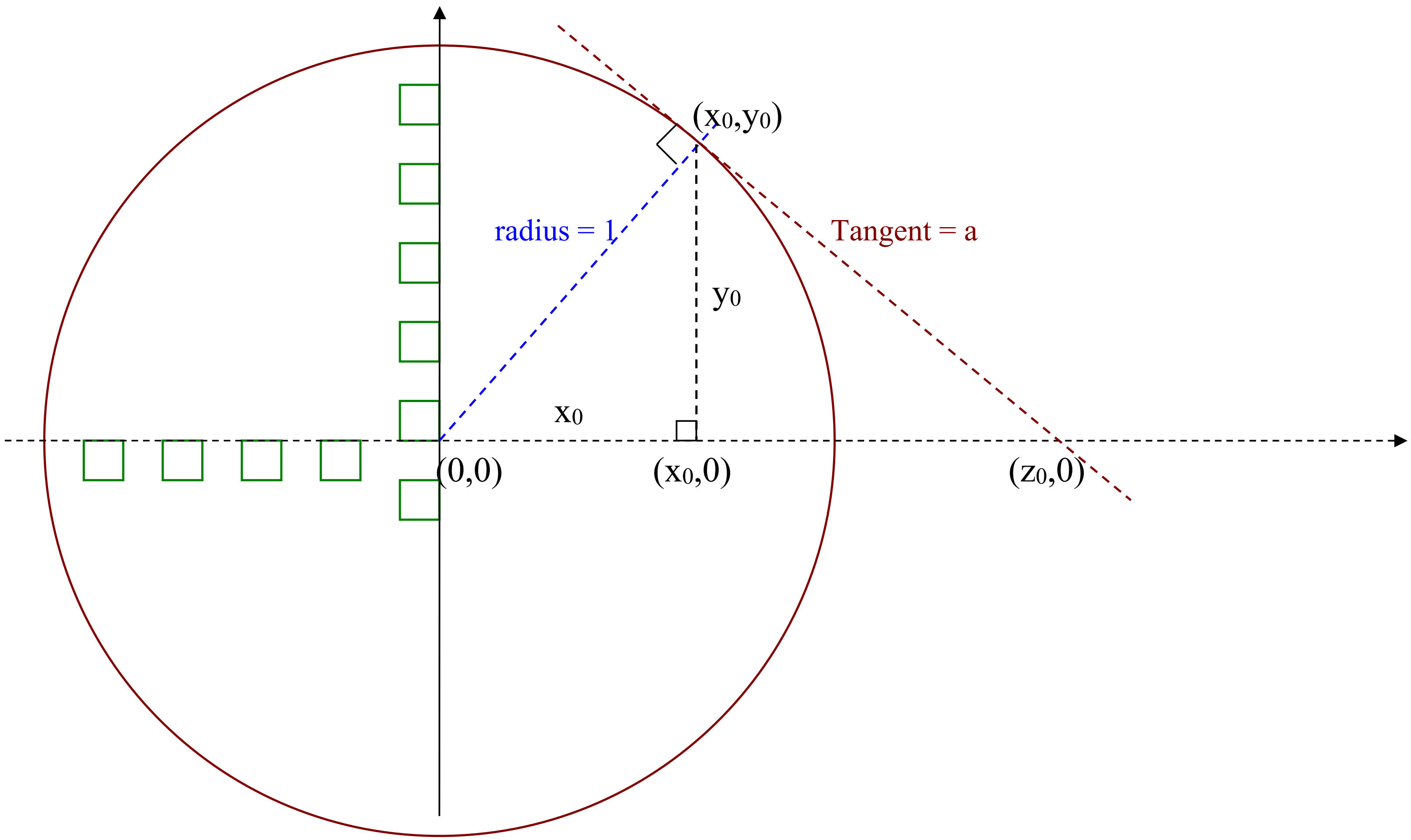

A Geometrical Representation of the Reciprocal Value of a Variable $z_0 = 1 / x_0$

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For a given value x_0 **of a variable x, the task is to find geometrically the reciprocal:**

$$z_0 = 1 / x_0$$

If the value $0 \le x_0 \le 1$ **then I use the following approach. Otherwise it will be shown later that a similar approach and same figure can be applied for values larger than 1.**

A unit circle, i.e. with radius = 1 needs to be drawn with its center at the origin (0,0) of the Cartesian x-y coordinates. Then the point (x_0,y_0) **is on the unit circle's circumference, where**: $y_0 =$ **sqr root of** $(1 - x_0^2)$ **as it is known that** $x_0^2 + y_0^2 = 1$

The tangent to the circle at the point (x_0,y_0) **is a straight line perpendicular to the radial line from** $(0,0)$ **to** (x_0,y_0) **and is intersecting the x-axis at the point** $(z_0,0)$. **Now it's time to prove that:** $z_0 = 1 / x_0$

Let "a**" denotes the length of the tangent segment between points** (x_0,y_0) **and** $(z_0,0)$. **As shown in the figure there are three right triangles. Then obviously:**

$x_0^2 + y_0^2 = 1$, $a^2 = y_0^2 + (z_0 - x_0)^2$ **and** $z_0^2 = a^2 + 1$

Solving for $a^2 = y_0^2 + (z_0 - x_0)^2 = y_0^2 + z_0^2 - 2\, z_0\, x_0 + x_0^2 = 1 + z_0^2 - 2\, z_0\, x_0$ **and hence:**

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$z_0^2 = 1 + z_0^2 - 2\, z_0\, x_0 + 1$ **and then** $0 = 2 - 2\, z_0\, x_0$ **leading to:** $2 = 2\, z_0\, x_0$ **or** $1 = z_0\, x_0$ **and finally:**

$$z_0 = 1/x_0$$

For a given value larger than 1 which can be represented by the x-coordinate of a point $(z_0,0)$ **outside the circle, the reciprocal will be** $x_0 = 1/z_0$ **using the same approach and figure.**