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Right-angled Triangle Calculation

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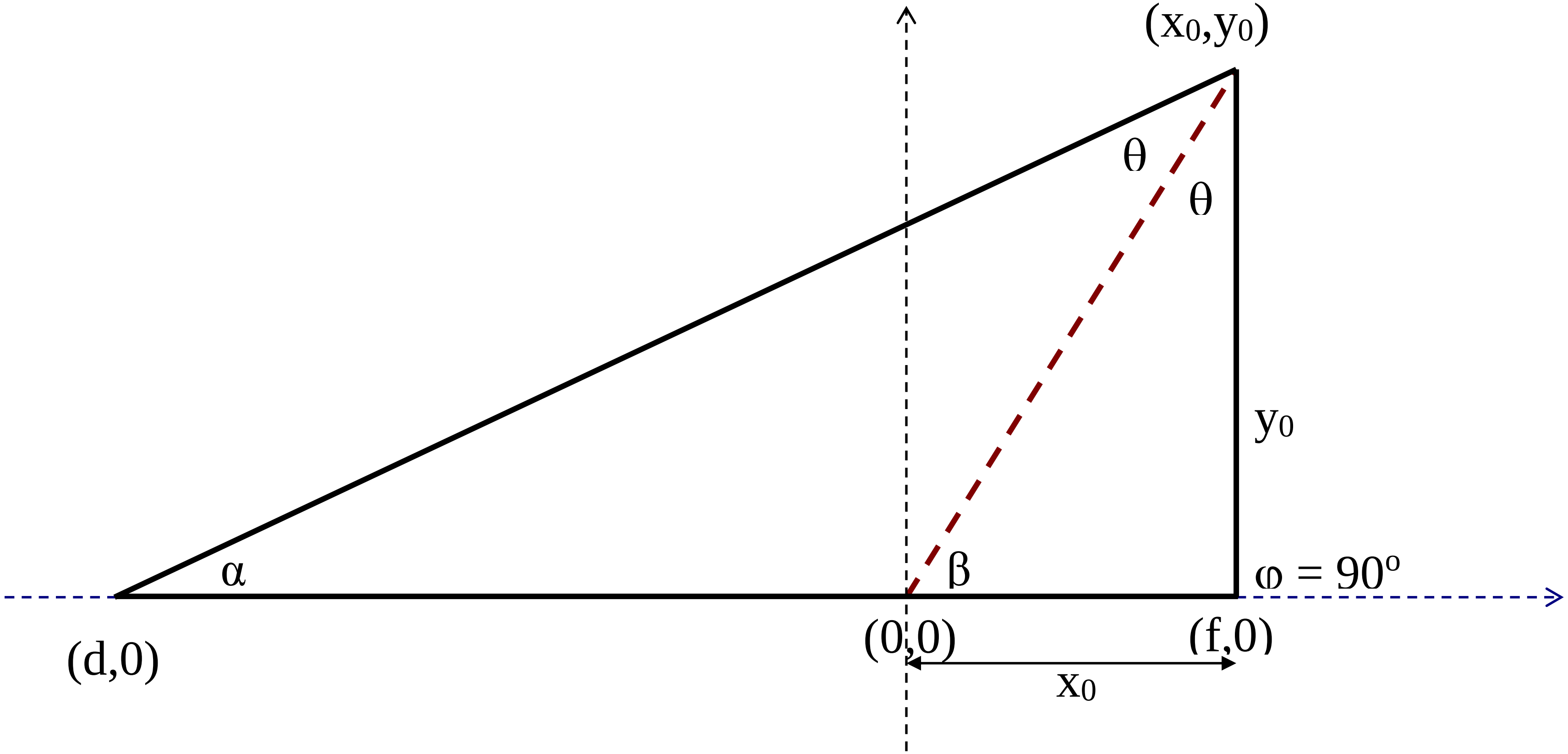

The following calculation is an extension of the obtuse triangle calculation. The triangle in figure is typical right-angled one i.e. the exterior angle φ is equal to 90º.

As before the base (of the altitude y_0) is chosen to be on the x-axis. The bisector (the red dashed line) of the vertex angle intersects with the base at the origin point (0,0) of the Cartesian x-y coordinates. As a result, the base of the triangle is consisting of two parts f **and** d **and it should be clear and obvious that** $f = x_0$ **in this scenario. The task is proposed to find the relationship between these two parts in this particular case. Here is the approach!**

It will be more convenient to avoid using the angle φ in calculations due to the fact that $\tan(\varphi)$ = infinity compared with the scenario presented for the obtuse triangle. Also it will be shown that the tangents of the angles θ, β and α are sufficient to find out the relationship between the two parts of the base, since it's possible to eliminate the altitude y_0 from the used equations. According to the figure of this scenario:

$\tan(\beta) = y_0 / x_0$, $\tan(\theta) = x_0 / y_0$ and $\tan(\alpha) = y_0 / (x_0 - d)$, d here is negative!

The exterior angle $\beta = \theta + \alpha$, and this yields to:

$\tan(\beta) = \tan(\theta + \alpha)$, and by applying the tangent of sum:

$$\tan(\beta) = \frac{\tan(\theta) + \tan(\alpha)}{1 - \tan(\theta)\tan(\alpha)}$$

The above equation can be simplified by multiplying the numerator of the left side with the denominator of the right side which results another equation:

$[\tan(\beta)]\,[1 - \tan(\theta)\tan(\alpha)] = [\tan(\theta) + \tan(\alpha)]$

Prior continue it is important to be very careful with the +/-- signs. Now substitute the tangents by their values:

$$[\, y_0 / x_0 \,]\,[\, 1 - (x_0 / y_0)\{y_0 /(x_0 - d)\} \,] = [\, (x_0/ y_0) + (y_0 / (x_0 - d)) \,]$$

We can rewrite the above equation in order to recognize common values:

$$\frac{y_0}{x_0}\left[1 - \frac{x_0\, y_0}{y_0\,(x_0 - d)}\right] = \left[\frac{x_0}{y_0} + \frac{y_0}{(x_0 - d)}\right]$$

You can notice that y_0 can be cancelled by dividing by y_0 and we get:

$$\frac{y_0}{x_0}\left[1 - \frac{x_0}{(x_0 - d)}\right] = \left[\frac{x_0}{y_0} + \frac{y_0}{(x_0 - d)}\right]$$

Further simplifying:

$$\frac{y_0}{x_0}\left[\frac{(x_0 - d) -- x_0}{(x_0 - d)}\right] = \left[\frac{x_0 (x_0 - d) + y_0^2}{y_0 (x_0 - d)}\right]$$

Multiplying both sides by $(x_0 - d)$ as well as eliminating x_0 will result:

$$\frac{y_0}{x_0}[-- d] = \left[\frac{-- (d x_0) + x_0^2 + y_0^2}{y_0}\right]$$ and subsequently:

$$y_0^2 (-- d) = -- (d x_0^2) + x_0 (x_0^2 + y_0^2)$$

Since $(x_0^2 + y_0^2) = r^2$ i.e. the square of the *length* of the bisector (the red dashed line) which is denoted by the letter r, the last equation will become simpler:

$$(-- d)(r^2 -- x_0^2) = -- (d x_0^2) + r^2 x_0$$

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$2d\,x_0^2 - d\,r^2 = r^2 x_0$ and then: $d\,(2x_0^2 - r^2) = r^2 x_0$

$$d = \frac{r^2 x_0}{(2x_0^2 - r^2)}$$ which can be rewritten as:

$$d = \frac{x_0}{2(x_0/r)^2 - 1}$$, put it another way using the reciprocal:

$$\frac{1}{d} = 2(x_0/r^2) - \frac{1}{x_0}$$, or $$\frac{1}{d} + \frac{1}{x_0} = 2(x_0/r^2)$$,

then by putting $x_0 = f$ on the left side we get:

$$\frac{1}{d} + \frac{1}{f} = 2\,x_0/r^2$$

However this is exactly the same relationship as with the obtuse triangle but here $f = x_0$ when the exterior angle φ is equal to 90º, i.e. the triangle is right-angled in this case. Referring to this equation:

$d(2x_0^2 -- r^2) = r^2 x_0$ we can solve for x_0: and consequently for f:

$x_0^2 -- (r^2/2d)\, x_0 -- (r^2/2) = 0$ which has the roots:

$x_0 = (r^2/4d) \pm [\, (r^2/4d)^2 + (r^2/2)\,]^{1/2}$ and we choose the + value as shown in figure:

$f = x_0 = (r^2/4d) + [\, (r^2/4d)^2 + (r^2/2)\,]^{1/2}$, I'll be discussing the -- value in future articles.